Table 1. Summary of test program.

Series	No. of Specimens	Type of Concrete	Horizontal Stirrups	Design N_u/V_u	Design v_u(psi)
A	2	Sand & Gravel	No	0	800
B	4	Sand & Gravel	Yes	0	800
C	4	Sand & Gravel	Yes	0.75	800
D	3	Sand & Gravel	Yes	1.0	565
E	4	Sand & Gravel	Yes	1.0	800
F	4	All-lightweight	Yes	1.0	800
G	1	All-lightweight	Yes	0	800
H	5	Sand & Gravel	Yes	1.0	1200
J	1	All-lightweight	Yes	1.0	560

stress was made equal to $0.2f'_c = 800$ psi the maximum allowable according to the shear-friction provisions of Section 11.15 of ACI 318. In Series D and E, the design N_u/V_u was made equal to the proposed maximum value of 1.0 for this method of design. Within each of these tests series, a/d was varied, having values up to and including 1.0.

Series F, G and J were tested to check the design method proposed when applied to corbels made of all-lightweight concrete, subject to both vertical and horizontal loads. In these series also, the limiting values of v_u, N_u/V_u, and a/d were explored.

Series H was tested to examine the possibility of using the following previously proposed[7] equation for the shear transfer strength of sand and gravel concrete, in place of the shear-friction equation when designing corbels as proposed.

$$v_u = 0.8\,\rho f_y + 400 \text{ psi} \qquad (16)$$

but not greater than $0.3f'_c$.

The specimens were designed for v_u equal to the maximum value of $0.3f'_c$ simultaneously with N_u/V_u having its maximum value of 1.0 and a/d having values up to and including 0.68.

Grade 60 reinforcement was used in Series E through J, to ensure that any conclusions reached would be valid for corbels reinforced with this higher strength steel.

Test specimens

Each specimen consisted of a 38-in. length of column with two corbels projecting from the column in a symmetrical fashion, as shown in Fig. 3. The main tension reinforcement consisted of parallel straight bars, anchored by short bars of equal diameter welded across their ends. The closed horizontal stirrups were #2 deformed bars and were uniformly distributed in the two-thirds of the effective depth nearest the main tension reinforcement.

Steel bearing plates (4 x 1 x 6 in.) were welded to the main tension reinforcement. Series A, B, and G specimens had plain bearing plates. The remainder of the specimens had grooved bearing plates, as shown in Fig. 3, to facilitate applying the horizontal force to the corbels.

The amount and strength of reinforcement used in each specimen are shown in Tables 2 and 3. The design compressive strength of the concrete at test was 4000 psi for all specimens. The actual concrete strengths at test are given in Tables 2 and 3.

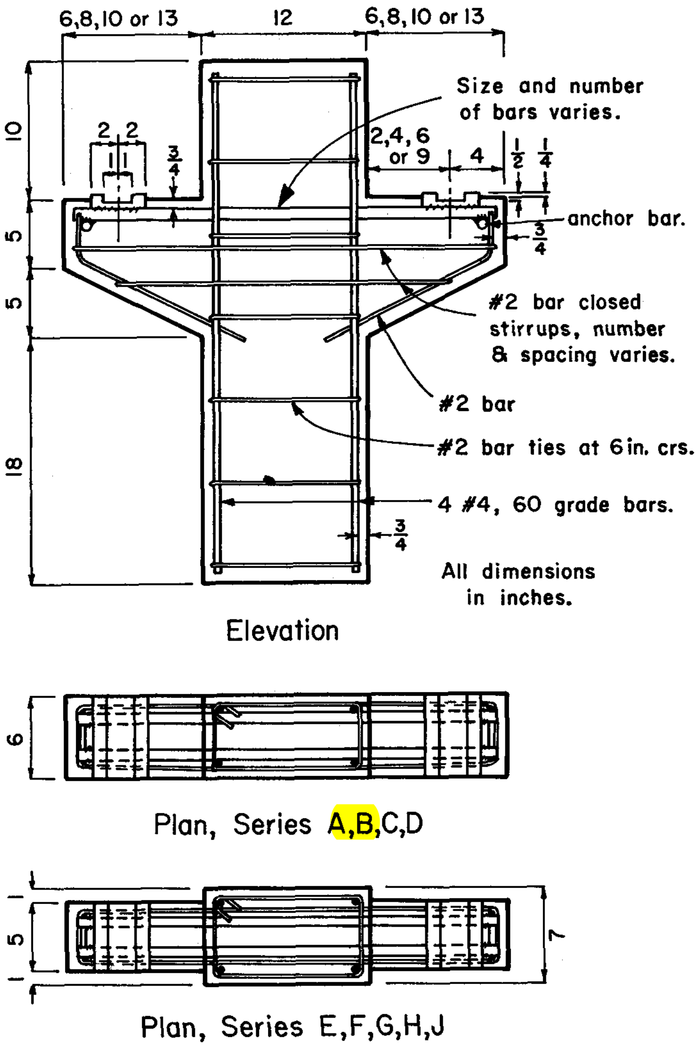

Fig. 3. Typical corbel test specimens.

Design of test specimens

The test specimens were designed as proposed above, except that in the case of Series C and D, the term $N_u(h\text{-}d)$ was omitted when calculating A_s using Eq. (5). This was done because it was initially assumed that the unconservatism of this action would be cancelled by the conservatism of ignoring the resistance moment contribution of the horizontal stirrups. However, when the test results from Series C and D were studied, this assumption was found to be incorrect. The term $N_u(h\text{-}d)$ was therefore included when calculating A_s in the design of the specimens of Series E through J. The capacity reduction factor was taken equal to 1.0 in these design calculations.

Series A—The specimens of Series A

were intended to provide a reference as to the effectiveness of the horizontal stirrups provided in the specimens of Series B. The specimens of Series A were therefore made as nearly as possible identical to the corresponding specimens of Series B, except that the horizontal stirrups were omitted.

Series B—All the specimens of Series B were designed to have an ultimate shear stress of $0.2f'_c$, i.e., 800 psi. The first three specimens of the series were provided with minimum stirrup reinforcement as proposed above, that is:

$$\rho_h f_{vy}\,(\text{min.}) = v_u - 150\left(f'_c\,\rho\,\frac{d}{a}\right)^{1/3} \tag{15}$$

The value of $\rho_h f_{vy}$ (min.) calcuated using Eq. (15) was found to be almost a constant quantity, being 186, 183, and 179 psi for Specimens B1, B2, and B3, respectively. This is very nearly equal to $\frac{1}{3}(A_{vf}f_{vy}/bd) = 190$ psi in this case where $v_u = 800$ psi.

This is apparently due to the occurrence of the product $(\rho \cdot d/a)$ in the equation for v_{u2}. When the shear span "a" is increased, then for a constant V_u, M_u increases and "ρ" increases almost in proportion to the increases in "a." Hence the product $(\rho \cdot d/a)$ remains approximately constant and therefore v_{u2} remains approximately constant.

For smaller values of v_u, ρ would decrease in proportion to v_u but v_{u2} would decrease less rapidly because it is proportional to $\rho^{1/3}$. The quantity $(v_u - v_{u2})$ would therefore decrease more rap-

Table 2. Details of corbel specimens (Series A, B, C & D).

Spec. No.	$\frac{a}{d}$	Main Reinforcement: Bars	Main Reinforcement: A_s (in.^{2})	Main Reinforcement: f_y (ksi)	Stirrups: A_h (in.^{2})	Stirrups: f_{vy} (ksi)	$\rho = \frac{A_s}{bd}$	Concrete Strength f'_c (psi)
A2	0.67	2 #5	0.62	46.6	0	-	0.0116	3675
A3	1.01	2 #6 +1 #3	0.99	54.1 53.6	0	-	0.0186	3850
B1	0.44	2 #4	0.40	48.5	0.20	65.0	0.0075	3630
B2	0.67	2 #5	0.62	46.5	0.20	67.0	0.0116	3450
B3	1.01	2 #6 +1 #3	0.99	54.1 53.2	0.20	64.0	0.0186	3760
B3A	1.01	2 #6 +1 #3	0.99	52.5 50.9	0.40	65.5	0.0186	4165
C1	0.45	2 #6 +1 #4	1.08	50.0 48.3	0.20	60.0	0.0203	4010
C2	0.68	3 #6	1.32	50.4	0.20	67.4	0.0248	3715
C2A	0.68	2 #6 +1 #5	1.19	50.0 47.4	0.20	65.6	0.0223	3705
C3	1.02	2 #7 +1 #6	1.64	50.0 50.9	0.40	65.5	0.0310	4385
D1	0.45	2 #6	0.88	50.2	0.10	70.0	0.0165	3910
D2	0.68	2 #6 +1 #4	1.08	47.7 47.5	0.20	64.0	0.0203	3805
D3	1.01	3 #6	1.32	48.4	0.25	67.6	0.0248	3700

idly than would ($v_u/3$) and hence, the minimum A_h by this hypothesis would be less than $A_{vf}/3$. At this point, it appeared that a simple and conservative rule for minimum stirrup reinforcement might be that it should be not less than $A_{vf}/3$.

Specimens B1 and B2 performed in a satisfactory manner, but Specimen B3 developed an ultimate strength of only 600 psi. The characteristics of the specimens as constructed were re-examined. It was found that because the actual strength of the larger diameter bars used for A_f was less than the value of 50 ksi assumed in the design and the strength of the #2 bars used for the horizontal stirrups was more than 50 ksi, the yield strength of the stirrups provided in Specimen B2 was very close to $A_f f_y/2$.

A fourth specimen, B3A, was therefore designed in which A_f was made the same as in Specimen B3, but a larger number of stirrups was provided so that their total yield strength $A_h f_{vy}$ was as nearly as possible equal to $A_f f_y/2$. This specimen behaved in a satisfactory manner.

It therefore appeared that it might be appropriate to require that the closed stirrups or ties parallel to the main tension reinforcement shall have a total yield strength at least equal to half the

Table 3. Details of corbel specimens (Series E, F, G, H & J).

Spec. No.	$\frac{a}{d}$	Main Reinforcement Bars	A_s (in.^{2})	f_y (ksi)	Stirrups A_h (in.^{2})	f_{vy} (ksi)	$\rho = \frac{A_s}{bd}$	Concrete Strength f'_c (psi)
E1	0.22	1 #6 +2 #4	0.84	59.1 67.5	0.15	64.0	0.0189	4030
E2	0.45	2 #6 +1 #2	0.93	59.7 67.0	0.15	67.0	0.0210	4450
E3	0.68	2 #6 +1 #4	1.08	62.5 64.0	0.25	65.0	0.243	4220
E4	1.01	3 #6	1.32	62.5	0.35	67.0	0.0297	4055
F2	0.45	2 #6 +1 #2	0.93	62.5 65.0	0.15	68.2	0.0210	3715
F3	0.68	2 #6 +1 #4	1.08	62.3 64.0	0.25	64.0	0.0243	3730
F4	1.01	3 #6	1.32	63.2	0.35	68.2	0.0297	4035
F4A	1.01	3 #6	1.32	63.4	0.35	68.0	0.0297	3715
G4	0.99	1 #6 +2 #3	0.66	62.5 65.5	0.30	67.4	0.0147	3750
H1	0.23	2 #6 +1 #2	0.93	64.8 68.2	0.20	64.0	0.0210	3920
H2	0.45	2 #6 +1 #4	1.08	62.4 64.0	0.25	64.0	0.0243	3930
H3	0.68	3 #6	1.32	63.2	0.35	67.0	0.0297	3855
H3A	0.68	3 #6	1.32	64.1	0.35	68.6	0.0297	3960
H3B	0.68	2 #6 +1 #6	1.32	62.5 64.8	0.35	68.0	0.0297	3820
J4	1.01	2 #6	0.88	64.8	0.25	64.0	0.0198	3645

yield strength of the reinforcement required for flexure, or one-third the yield strength of the reinforcement required to resist shear (calculated using the shear-friction provisions of ACI 318-71), whichever is the greater.

If the yield point of the stirrups and the main tension reinforcement is the same, the above requirement reduces to $A_h(\text{min.}) = A_f/2$ or $A_{vf}/3$, whichever is the greater.

Series C—The corbels of this series were designed to be companion specimens to Corbels B1, B2, and B3A. The Series C corbels were provided with additional main tension reinforcement, so that the strength of the main tension reinforcement in the Series C corbel was greater than that provided in its companion Series B corbel by an amount equal to the horizontal force N_u that it was proposed to apply to the corbel concurrently with the shear.

The horizontal stirrup reinforcement was made the same as that provided in the companion Series B corbels. That is, $A_h f_{vy}$ was made as nearly as possible equal to the greatest of the values given by $\frac{1}{3} A_{vf} f_{vy}$ or $\frac{1}{2} A_f f_y$.

This was done to check whether this amount of stirrup reinforcement is adequate when the maximum allowable horizontal force acts on the corbel concurrently with the shear. These corbels behaved satisfactorily. Therefore, this amount of stirrup reinforcement was provided in all subsequent test specimens.

Series D and E—The specimens of both these series were designed to carry an ultimate horizontal force equal to the ultimate shear force. However, the corbels of Series E were made of a reduced width, so that a design ultimate shear stress of 800 psi could be attained when using available hydraulic rams to apply the horizontal force.

Series F, G, and J—The specimens of Series F were of all-lightweight concrete and were designed to have the same ultimate shear strength and ultimate outward horizontal force as the corresponding specimens of Series E. However, in this case the shear-friction calculation was modified as proposed previously,[6] μ being multiplied by the coefficient 0.75 for all-lightweight concrete contained in Section 11.3.2 of ACI 318-71.

Specimen F2 ($a/d = 0.45$) behaved satisfactorily, yielding a v_u of 825 psi. However, Specimen F4 ($a/d = 1.01$), which was tested next, yielded a v_u of only 540 psi, or $0.13f'_c$. The characteristics of the specimen as constructed were re-examined, but there was no immediately apparent reason for the poor behavior. A duplicate specimen F4A was fabricated and tested. It behaved the same as Specimen F4, yielding a v_u of $0.14f'_c$. A specimen G4, with the same a/d as Specimen F4, was designed to carry an ultimate shear stress of $0.2f'_c$ with zero horizontal load. It was subjected to vertical load only, to determine whether the presence of the horizontal load N_u had influenced the maximum shear stress attained in Corbels F4 and F4A.

The ultimate shear stress developed in Specimen G4 was also $0.14f'_c$, indicating that the unexpected low value of v_u in Specimens F4 and F4A did not result from the action of N_u. Neither the main tension reinforcement nor the stirrup reinforcement developed their yield strengths in Corbels F4, F4A, and G4. These corbels appeared to fail by shear-compression of the concrete in a beam type shear failure.

In push-off tests of all-lightweight concrete previously reported[6] (i.e., $a/d = 0$), it was possible to develop a maximum shear transfer stress of $0.2f'_c$. It is apparently necessary to define the upper limit of v_u for structural lightweight concrete corbels in terms of a/d. It was decided to check whether a shear stress of $0.14f'_c$ could still be developed at $a/d = 1.0$ if the main reinforcement yielded.

Specimen J4 ($a/d = 1.0$) was designed to have $v_u = 0.14f'_c$, $N_u/V_u = 1.0$ and a moment capacity (based on the reinforcement yield strength) corresponding to the design ultimate shear. This corbel developed an ultimate shear stress of $0.13f'_c$ and the reinforcement just reached its yield point at failure.

Series H—The sand and gravel concrete corbels of this series were designed to have an ultimate shear strength of 53.4 kips with $N_u/V_u = 0.68$. This corresponds to the upper limit shear stress of $0.3f'_c$ specified when using Eq. (16) for shear design, instead of the shear-friction equation.

Materials and fabrication

The sand and gravel concrete was made from Type III portland cement, sand, and ¾-in. maximum size gravel, in the proportions 1.0:2.77:3.44 by weight.

The all-lightweight concrete was made from the same coated aggregate used in the study[6] of shear transfer strength. The concrete was made from Type I portland cement, coarse and fine lightweight aggregate in the proportions 1.0:1.48:2.10. In both concretes, water was added sufficient to produce a 3-in. slump. Approximately 6 percent of air was entrained in the concrete. The sand and gravel concrete was moist cured 1 day and then cured in the air of the laboratory until test at age 4 days.

The lightweight concrete was moist cured 7 days and then cured in air until test at 28 days. (This is the standardized curing procedure for tests of structural lightweight concrete.)

The deformed reinforcing bars used for the main tension reinforcement and for the column reinforcement conformed to ASTM Specification A615. The #2 bars used for stirrup reinforcement had deformations similar to the larger bars conforming to ASTM Specification A615.

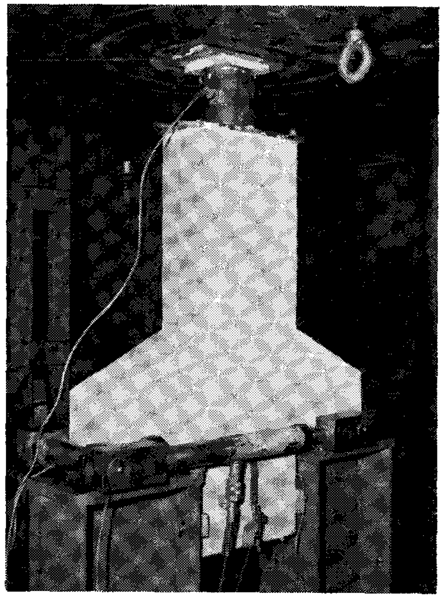

Fig. 4. Arrangement for test of corbel specimens.

A stiff metal jig was used to hold the bearing plates in alignment with one another and the correct distance apart, while they were being welded to the main tension reinforcement. In the form, the bearing plates were screwed to metal supports to ensure their correct alignment and location.

Testing arrangements and instrumentation

For convenience, the specimens were tested in an inverted position, as may be seen in Fig. 4. The specimens of Series A, B, and G were supported directly on the plain bearing plates, through free rollers resting on the tops of the two legs of the U-shaped supporting frame. The vertical load was applied by a Baldwin hydraulic testing machine through an SR4 gage load cell located concentrically on the top of the column.

The specimens of Series C, D, E, F, H, and J were also supported on free rollers; but through an intermediate bearing plate having a projection on its upper face which mated with the ½ in. deep groove in the bearing plate welded to the corbel reinforcement. Short stub axles projected from each end of the intermediate bearing plates, the centers of the stub axles being at the level of the horizontal face of the corbel.

The horizontal force was applied to the corbels through these stub axles by two 20-kip capacity hydraulic rams, arranged one in front and one behind the specimen, as may be seen in Fig. 4. The horizontal force was monitored by two links instrumented with SR4 gages so as to act as transducers. The vertical load was applied by the testing machine in the same way as for Series A and B.

The average deflection of the corbels at their loading points was measured using a linear differential transformer, attached to the column at its centerline. Resistance strain gages were used to measure the strain in the main tension reinforcement at the interfaces between the column and the corbels in all tests.

In Series F, G, H, and J the strain in the stirrup reinforcement was measured at points where the stirrups crossed a line joining the center of the bearing plate and the intersection of the sloping face of the corbel and the column face. The load cell, transducer links, differential transformer, and strain gages were monitored continuously during the tests by a Sanborn strip chart recorder.

In Series E through J, the maximum width of crack was measured at an arbitrary "service load" of 0.55 V_u (design) and $0.5N_u$.

Testing procedure

The specimens of Series A and B were subjected to an incrementally increasing load until failure occurred.

The specimens of Series C and D were first subjected to the horizontal force N_u. This horizontal force was then maintained constant while the vertical load was increased incrementally, until failure occurred. It was considered that applying the full ultimate horizontal load before applying the vertical load was the most severe loading condition to which a corbel could be subjected, and that the measure of strength so obtained would therefore be conservative.

In the remaining tests, it was decided to try and obtain some measure of serviceability as well as strength, by measuring crack widths at service load. It was arbitrarily decided to consider that the dead load shear V_D and the live load shear V_L were equal. Then, if $\phi = 0.85$, we have $V_D = V_L = 0.275\,V_u$.

Also, since ACI 318-71 requires that the horizontal force be treated as a live load, the service horizontal force $N = 0.5N_u$. The following loading sequence was used:

(1) $N = 0$, V increased incrementally to $V_D = 0.275\,V_u$.

(2) $V = V_D$, N increased incrementally to $0.5\,N_u$.

(3) $N = 0.5N_u$, V increased incrementally to $V_D + V_L = 0.55\,V_u$.

(4) $V = V_D + V_L$, N increased incrementally to N_u.

(5) $N = N_u$, V increased incrementally until failure occurred.

The maximum crack widths at $V = V_D + V_L$ and $N = 0.5\,N_u$ were recorded, this being regarded as the service load.

In all the tests the cracks were marked at each increment of loading.

Test Results

Specimen behavior

Specimens subject to vertical load only—The first cracks to form were flexure cracks, which propagated from the intersection of the column face and the horizontal face of the corbel. These cracks penetrated about halfway through the depth of the corbel at about half the ultimate shear, at which time diagonal tension cracks appeared in the corbels.

These cracks were aligned roughly

along a line running from the intersection of the sloping face of the corbel and the column face, to a point between the inner edge of the bearing plate and the center of the bearing plate. The initial length of these cracks was about one-third the effective depth. As the applied shear was further increased, the diagonal tension cracks increased in length, at first rapidly, then much more slowly as the ultimate load was approached.

In the case of the corbels without stirrups, essentially only one diagonal tension crack formed in each corbel and a very sudden failure occurred when the concrete at the head of these cracks sheared through. This type of diagonal tension failure was referred to by Kriz and Raths[2] as a "diagonal splitting" failure.

In the case of the corbels with horizontal stirrups, additional inclined cracks occurred, usually starting as flexure cracks at the horizontal face of the corbel. The failure of Corbel B1 was classified as a flexural failure, since it was characterized by wide opening of the flexural cracks, while the diagonal tension cracks remained fine.

The failures of the remaining corbels of Series B were classified as "beam shear" type failures, i.e., a shear failure of the type which occurs in a reinforced concrete beam with stirrup reinforcement. In this case, the flexure cracks remained fine and failure was characterized by widening of one or more diagonal tension cracks and the shear-compression failure of the concrete near the intersection of the sloping corbel face and the column face. Failure was quite abrupt, but less brittle and with more warning than in the case of the diagonal tension failures of the corbels without stirrups. In all cases, the deflections were very small and provided no warning of failure.

Specimens subject to both vertical and horizontal loads—Two types of crack were caused by horizontal force:

(1) More or less vertical cracks due to the direct tension stresses produced in the concrete by the horizontal force, and

(2) Cracks approximately aligned with the main tension reinforcement and evidently caused by the splitting action of the bar deformations due to the high bond stresses caused by the transfer of some of the horizontal force from the main tension reinforcement to the surrounding concrete.

The first cracks to form due to vertical loads were flexure cracks, which either occurred independently or as extensions of the more or less vertical cracks caused by the horizontal force. Diagonal tension cracks initiated at between 40 and 75 percent of the ultimate load, the average diagonal tension cracking shear being 58 percent of ultimate. In some cases, they propagated from the end of a vertical crack caused by the horizontal force, but in other cases, they initiated independently and cut across existing vertical cracks caused by the horizontal forces.

A typical example of the development of cracks is shown in Fig. 5. The dashed lines indicate cracks which occurred when the horizontal load was increased. Full lines are cracks which occurred when the vertical load was increased.

Corbel Specimen D1 failed in flexure with wide opening of the flexure crack adjacent to the column face as the main tension reinforcement yielded, the diagonal tension cracks remaining fine. The failures of all the other specimens subject to both vertical and horizontal forces (except Specimen E4) were classified as "beam shear" type failures and were characterized by widening of the diagonal tension cracks at failure and shear compression failure of the concrete adjacent to the intersection of the sloping face of the corbel and the column face.

shear force V_u (test) are shown in Tables 4 through 8, together with the nominal ultimate shear stress v_u (test) = V_u (test)/bd and the shear force V_y (test) which was acting at yield of the main reinforcement. The type of failure exhibited by each specimen is also indicated in these tables as follows:

D.T. = diagonal tension failure
F = flexural failure
B.S. = "beam shear" type failure

Discussion of test results

Minimum stirrup reinforcement—The brittle and complete diagonal tension failure of the Series A corbels without stirrups, provides additional support for Kriz and Raths' contention that all corbels should be reinforced with horizontal stirrups in addition to the main tension reinforcement, in order to eliminate the possibility of this type of failure occurring.

Except for Specimen B3, the remaining corbels all had horizontal stirrups, the yield strength of which was half the yield strength of that part of the main tension reinforcement not resisting the horizontal force N_u, i.e.,

$$A_h f_{vy} = \tfrac{1}{2}(A_s f_y - N_u)$$

In all these corbels which were made

Table 4. Test data—Series A & B (all forces in kips).

	Specimen No.	A2(1)	A3(1)	B1	B2	B3A
1.	N_u	0	0	0	0	0
2.	V_u(test)	35.6	28.0	47.0	38.9	42.1
3.	v_u(test)(psi)	664	526	870	725	791
4.	v_u/f'_c	0.18	0.14	0.24	0.21	0.19
5.	V_y(test)	*	*	39.7	+	*(2)
6.	V'_u(test)	35.6	28.0	39.7	38.9	42.1
7.	V_u(S.F.)	39.4	41.0	39.2	37.0	42.6
8.	V_u(Mod.S.F.)	44.5	61.5	47.5	54.9	66.6
9.	V(flex)	39.2	44.6	41.1	39.0	44.1
10.	V_1(calc)	39.2	41.0	39.2	37.0	42.6
11.	$\frac{V_u(\text{test})}{V_1(\text{calc})}$	0.91	0.68	1.20	1.05	0.99
12.	$\frac{V'_u(\text{test})}{V_1(\text{calc})}$	0.91	0.68	1.01	1.05	0.99
13.	V_2(calc)	39.2	44.6	41.1	39.0	44.1
14.	$\frac{V_u(\text{test})}{V_2(\text{calc})}$	0.91	0.63	1.14	1.00	0.95
15.	$\frac{V'_u(\text{test})}{V_2(\text{calc})}$	0.91	0.63	0.97	1.00	0.95
16.	Failure Type	D.T.	D.T.	F.	B.S.	B.S.

* Did not yield   + Strain at failure not known

(1) No stirrup reinforcement
(2) Main reinforcement very close to yield at failure

"D.T." denotes a diagonal tension failure
"F" denotes a flexural failure
"B.S." denotes a beam-shear type failure